

### Taylor Expansion

$$f(x) = \sum_{n=0}^{\infty} \left(\frac{1}{n!}\right) \cdot f^{(n)}(a) \cdot (x-a)^n$$
$$= f(a) + 1! \cdot f'(a) \cdot (x-a) + \frac{1}{2!} \cdot f''(a) \cdot (x-a)^2 + \frac{1}{3!} \cdot f^{(3)}(a) \cdot (x-a)^3 + \dots$$

### Approximations

$$\tan(\theta) \approx \theta \quad 1\% \text{ Relative Error at } \sim 0.176 \text{ radians}$$

$$\sin(\theta) \approx \theta \quad 1\% \text{ Relative Error at } \sim 0.244 \text{ radians}$$

$$\cos(\theta) \approx 1 - \frac{\theta^2}{2} \quad 1\% \text{ Relative Error at } \sim 0.664 \text{ radians}$$

$$\text{When } x \ll 1: \quad \frac{1}{1+x} \approx 1 - x$$

$$e^x \approx 1 + x$$

$$\ln(1+x) \approx x$$

### Fourth Order Runge-Kutta Method

$$y(x + \Delta x) \approx y(x) + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$k_1 = f(x, y)\Delta x$$

$$k_2 = f\left(x + \frac{1}{2}\Delta x, y + \frac{1}{2}k_1\right)\Delta x$$

$$k_3 = f\left(x + \frac{1}{2}\Delta x, y + \frac{1}{2}k_2\right)\Delta x$$

$$k_4 = f(x + \Delta x, y + k_3)\Delta x$$